

**CDF
CMEX
UPGRADE**

**STRUCTURAL
AND
MECHANICAL
DESIGN**

**ENTIRE
COMBINED
FRAME**

Entire Combined Frame

The CMEX inner, conical frame system is the platform on which the muon chambers are mounted. By itself, it is not strong enough nor does it minimize deflections needed for accurate data gathering. Therefore it was determined that an outer frame would be built and erected to support the inner, conical frame and chambers.

The outer frame, though strong enough to support all loads by itself, still yielded large deflections at the outermost point of around 3" in all X, Y, Z directions. When the inner, conical frame was added to the FE model, deflections at the outermost point dropped to values neighboring at .5" in all X, Y, Z directions. This was great; but unfortunately high stress concentrations popped up on the inner, conical frame in the region where the frame section modulus was smallest and bending moments largest. This was not great!

After a review of the situation, it was realized that a flaw in the FE model was contributing to unusual high stresses. The flaw was in letting the gravitational load of the outer support frame act down on the inner, conical frame. Before the inner frame is even added to the support frame, the support frame has deflected downward due to gravity. This means that the support frame has, in a sense, been pre-loaded and that its own weight has been negated from the problem and should therefore be assumed to be weightless as far as the FE model is concerned.

After removing the additional load on the inner frame caused by the weight of the support frame, the stresses in the critical region of the frame were reduced greatly. However, according to the FE program "Frame Analysis," these stresses were still above the allowable stress of 21,600psi.

After a period of time investigating the data and reviewing the "Frame Analysis" manual, it was discovered that combined stress was being calculated in a non-precise manner. The following equation (from Frame Analysis) is used to calculate combined stress:

$$\sigma_c = \sqrt{[\sigma_1 + \sigma_2 + \sigma]^2 + 3\tau_T^2}$$

Normal stress due to load along the X-axis of the beam element (same at all 4 locations)

P = axial load

$$\sigma = P/A$$

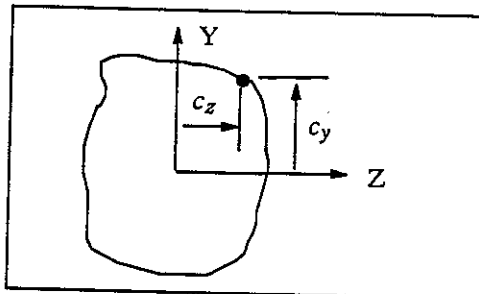
A = cross-section area

2. Normal stress due to bending about the Z-axis

$$\sigma_1 = \frac{M_z * C_y}{I_z}$$

3. Normal stress due to bending about the Y-axis

$$\sigma_2 = \frac{M_y * C_z}{I_y}$$



Maximum shear stress due to force in Y-direction (same at all 4 locations)

$$\tau_y = a_y \frac{F_y}{A}; a_y = \text{shear ratio in Y-direction}$$

Maximum shear stress due to force in Z-direction (same at all 4 locations)

$$\tau_z = a_z \frac{F_z}{A}; a_z = \text{shear ratio in Z-direction}$$

Shear stress due to torsion

$$\tau_T = \frac{T * r_{eff}}{K}$$

T = torque about the shear center axis

r_{eff} = effective radius in torsion

K = torsional constant

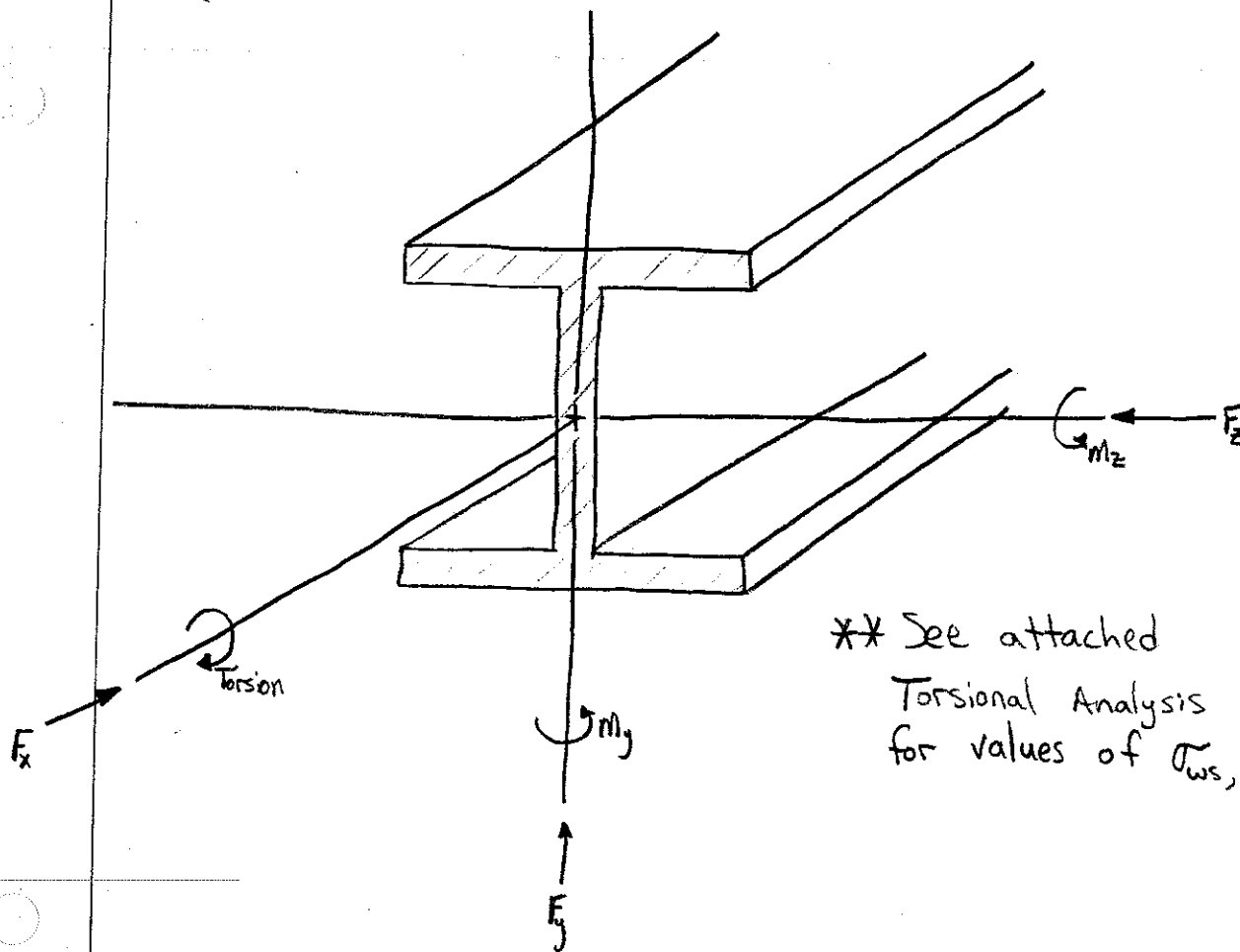
The problem lies in the handling of the torsion in a wide-flange member. Calculations to determine warping normal stress and web and flange shear stress due to torsion are complex and "Frame Analysis" chooses a simplified manner to add torsional stresses into the combined stress equation. This leads to over exaggerated high stresses for elements effected by torsion which, of course, is a common occurrence in a CMEX frame.

The solution was to take maximum forces and moments of all wide-flange members and assume them to all act on one member. Then calculate combined stress by hand and determine if it is below allowable limits.

Of course there still was the problem of calculating accurate induced stresses due to torsion on the wide-flange elements. This was handled easily by using an exact solution to a differential equation to solve for warping normal stresses, web shear stress, and flange shear stress. (equations and examples are attached) Once these stresses were known, they could be combined with bending stresses, shear stresses, and axial stress to obtain the stress vector magnitude; combined stress. The equation used is:

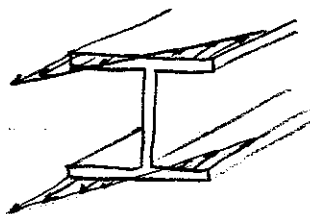
$$\sigma_c = \sqrt{(\sigma_x + \sigma_y + \sigma_z + \sigma_{ws})^2 + (\tau_y + \tau_f)^2 + (\tau_z + \tau_f)^2}$$

This is a much more reliable combined stress equation and since the maximum forces and moments were used in this equation and still produced a combined stress less than the allowable, it infers that all wide-flange members are adequate to support the system loads.



** See attached
Torsional Analysis
for values of σ_{ws} , γ_w , γ_f

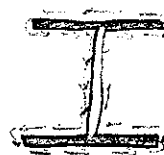
Torsional Stresses: (Numbers generated on MCAD, Torsional Analysis by Don Mitchell)



Warping Normal Stresses
(σ_{ws})



Web Shear Stress
(γ_w)
[small, may be
neglected]



Flange Shear Stress
(γ_f)

Standard Stresses:

Normal Stress

$$\frac{F_x}{A}, \frac{M_y}{S_y}, \frac{M_z}{S_z}$$

$$(\sigma_x), (\sigma_y), (\sigma_z)$$

Shear Stress

$$\frac{F_y}{A}, \frac{F_z}{A}$$

$$(\gamma_y), (\gamma_z)$$

Combined Stress:

$$\sigma_{max} = \sqrt{(\sigma_x + \sigma_y + \sigma_z + \sigma_{ws})^2 + (\gamma_y + \gamma_f)^2 + (\gamma_z + \gamma_f)^2}$$

To reduce the number of equations to solve, a conservative approach will be used.

Instead of calculating stresses for each wide-flange, the maximum forces and moments out of all the wide-flanges will be used and assumed to be acting all on one beam. If the combined stress is below allowable stress of 21,600 psi, then it can be assumed that all wide-flange elements are below allowable stress. If not, an individual analysis of each beam will be performed.

$F_{x_{max}}$	$F_{y_{max}}$	$F_{z_{max}}$	$m_{x_{max}}$	$m_{y_{max}}$	$m_{z_{max}}$
25210 lbs	3629 lbs	3252 lbs	14,270 in-lb torsion	24,070 in-lb	125,700 in-lb
σ_x	τ_y	τ_z	σ_{ws} τ_f	σ_y	σ_z
F_x/A	F_y/A	F_z/A		m_y/S_y	m_z/S_y
$\frac{25210}{7.08}$	$\frac{3629}{7.08}$	$\frac{3252}{7.08}$		$\frac{24070}{5.63}$	$\frac{125700}{20.9}$
3561 psi	513 psi	459 psi	5836 psi 1382 psi	4275 psi	6014 psi

Combined Stress:

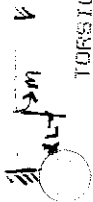
$$\sigma_{max} = \sqrt{(\sigma_x + \sigma_y + \sigma_z + \sigma_{ws})^2 + (\tau_y + \tau_f)^2 + (\tau_z + \tau_f)^2}$$

$$\sigma_{max} = \sqrt{(3561 + 4275 + 6014 + 5836)^2 + (513 + 1382)^2 + (459 + 1382)^2}$$

$$\sigma_{max} = 19,862 \text{ psi} \quad \underline{\underline{OK}} \quad F_a = .6 F_y = .6 (36,000 \text{ psi}) = 21,600 \text{ psi}$$

$$\underline{\underline{\sigma_{max} < F_a}}$$

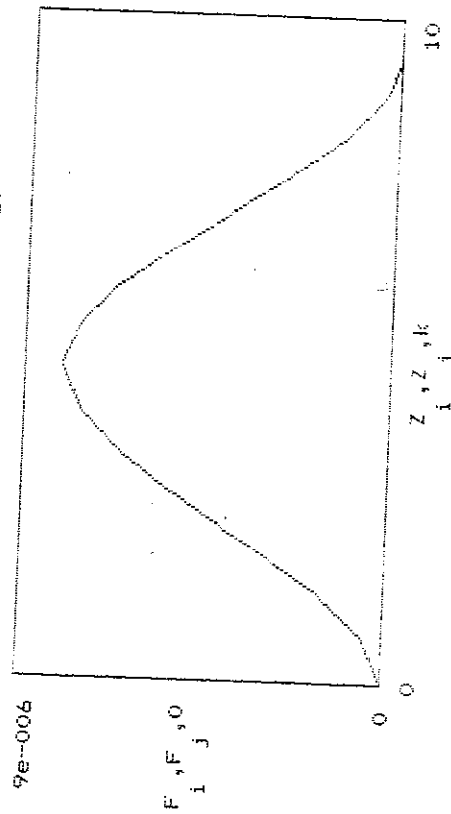
Therefore, it can be assumed that all wide-flange members are adequate.



DATA RESULTS:

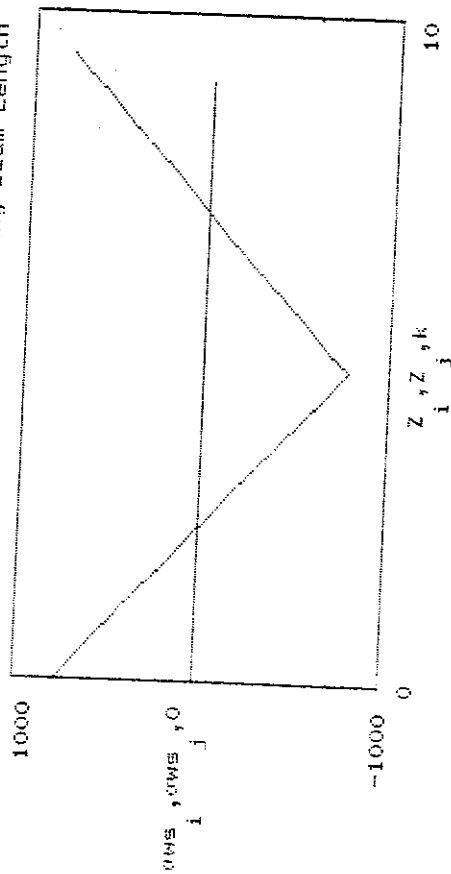
TWIST ANGLE AND ITS DERIVATIVES

Twist Angle (radians)



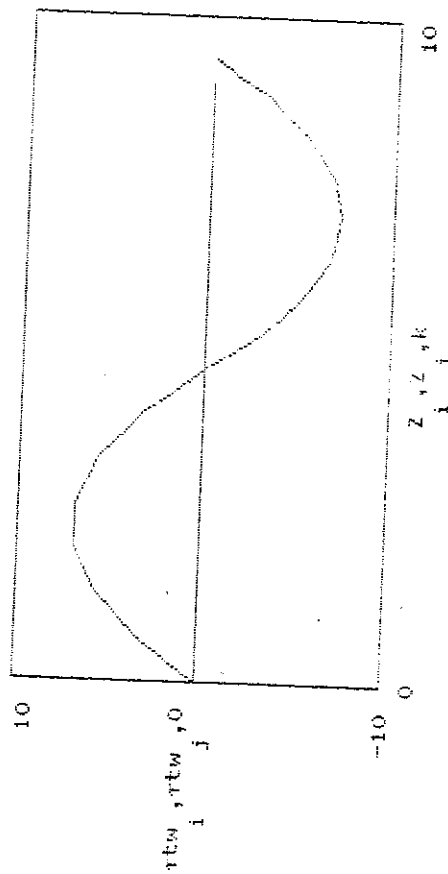
TORSIONAL STRESSES

Warping Normal Stresses Along Beam Length

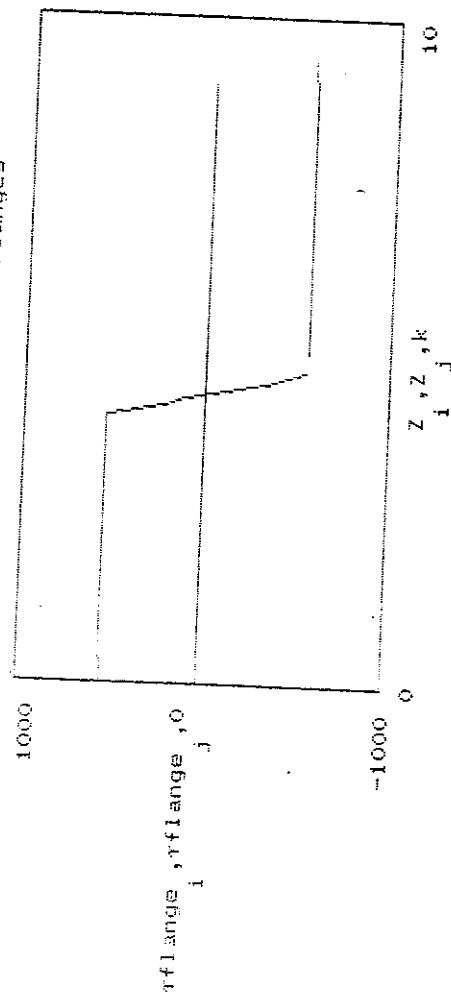


Torsional Shear Stresses Along Beam Length

Shear Stresses in Web



Shear Stress in Flanges



CASE 6

psi
Warping Normal
Stress
σ_{ws}

7.784 · 10 ⁻²
5.559 · 10 ⁻²
3.335 · 10 ⁻²
1.112 · 10 ⁻²
-3.335 · 10 ⁻²
-5.559 · 10 ⁻²
-7.784 · 10 ⁻²
-5.837 · 10 ⁻²
-3.891 · 10 ⁻²
-1.945 · 10 ⁻⁶
2.119 · 10 ⁻²
1.945 · 10 ⁻²
3.891 · 10 ⁻²
5.837 · 10 ⁻²
7.784 · 10 ⁻²

psi
Web Shear
Stress
τ_{tw}

3.014 · 10 ⁻¹²
3.463
5.771
6.925
6.925
5.771
3.463
-10
-3.093
-5.303
-6.528
-7.07
-6.528
-5.303
-3.093
-2.572 · 10 ⁻¹⁰

psi
Flange Shear
Stress
τ_{flange}

5.412 · 10 ⁻²
5.465 · 10 ⁻²
5.5 · 10 ⁻²
5.518 · 10 ⁻²
5.5 · 10 ⁻²
5.465 · 10 ⁻²
-5.412 · 10 ⁻²
-5.459 · 10 ⁻²
-5.493 · 10 ⁻²
-5.513 · 10 ⁻²
-5.52 · 10 ⁻²
-5.513 · 10 ⁻²
-5.493 · 10 ⁻²
-5.459 · 10 ⁻²
-5.412 · 10 ⁻²

Radians
Twist Angle
F

0
4.458 · 10 ⁻⁷
1.595 · 10 ⁻⁶
3.167 · 10 ⁻⁶
4.88 · 10 ⁻⁶
6.452 · 10 ⁻⁶
7.602 · 10 ⁻⁶
8.047 · 10 ⁻⁶
7.702 · 10 ⁻⁶
6.79 · 10 ⁻⁶
5.501 · 10 ⁻⁶
4.024 · 10 ⁻⁶
2.545 · 10 ⁻⁶
1.257 · 10 ⁻⁷
3.458 · 10 ⁻⁷
0

Position
Z

0
0.669
1.339
2.008
2.677
3.346
4.016
4.685
5.271
6.442
7.028
7.613
8.199
8.784
9.37

$$E = 2.9 \cdot 10^7$$

Modulus of Elasticity

$$G = 1.12 \cdot 10^7$$

Modulus of Rigidity

$$P = 2.521 \cdot 10^4$$

Axial Load (lb)

$$M = 1.427 \cdot 10^4$$

Torsional Moment (in-lb)

Wide-Flange Constants (from AISC handbook)

$$J = 0.35$$

Torsional Constant

$$a = 4.4 \cdot 10^1$$

(ECw/GJ)

$$W_{90} = 1.22 \cdot 10^1$$

Normalized Warping Constant

$$S_w = 7.94$$

Warping Static Moment

$$A = 7.08$$

Area of Cross-Section

Wide-Flange Dimensions (inches)

$$L = 9.37$$

Unsupported Length of Beam

$$t_w = 0.245$$

Web Thickness

$$t_f = 0.4$$

Flange Thickness

$$h = 7.93$$

Overall Height of Wide-Flange

$$w = 6.495$$

Overall Width of Wide-Flange

NOTE!!!!

All of the calculations are complete now. Hit the ESC key and type "GOTO" and hit the RETURN key. An

Case 9

WALL THICKNESS

1.5 in

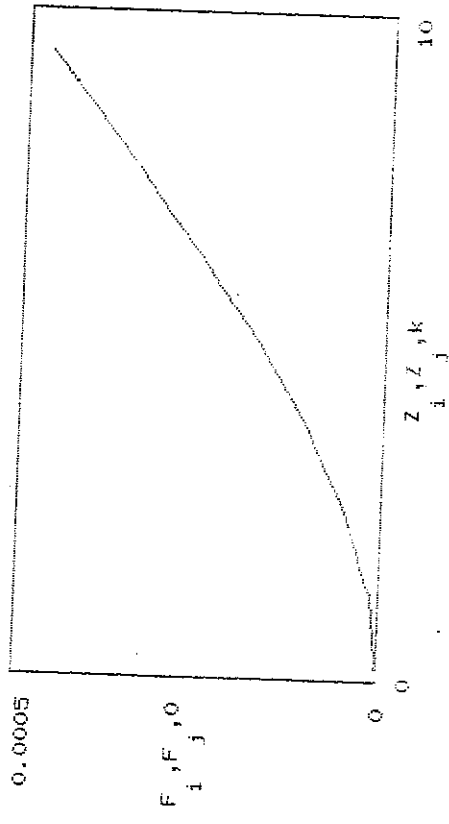
TORSIONAL ANALYSIS OF WIDE-FLANGE MEMBER

$\alpha = 0.95$

DATA RESULTS:

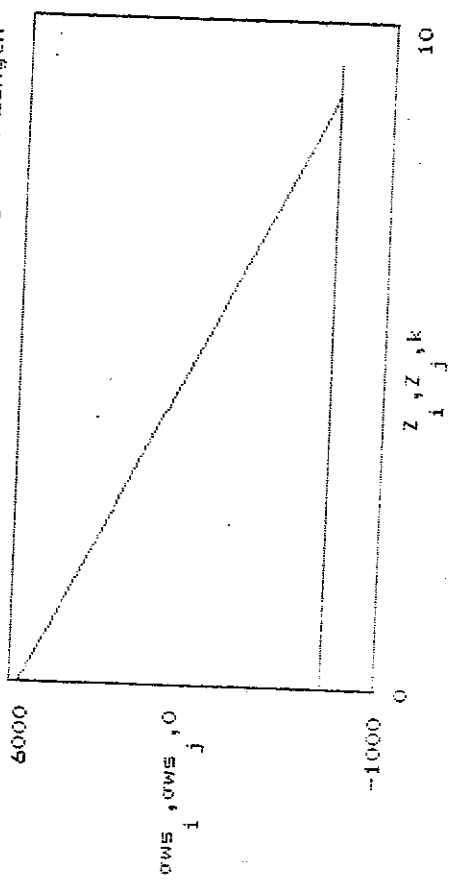
TWIST ANGLE AND ITS DERIVATIVES

Twist Angle (radians)



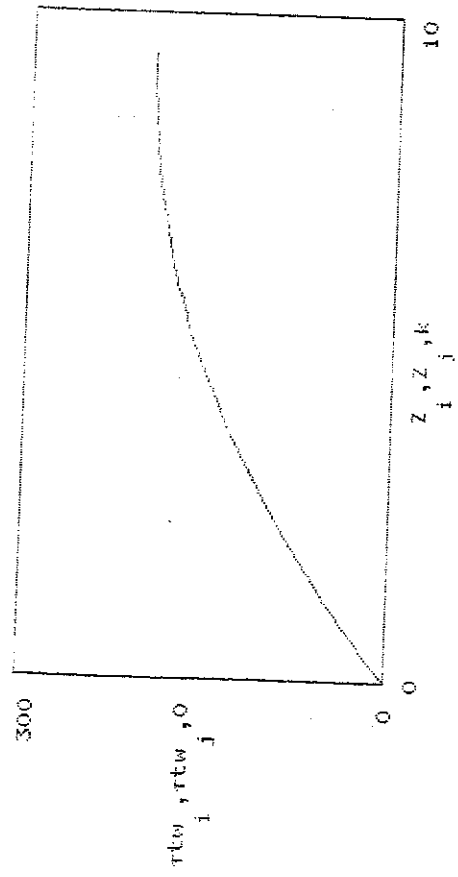
TORSIONAL STRESSES

Warping Normal Stresses Along Beam Length

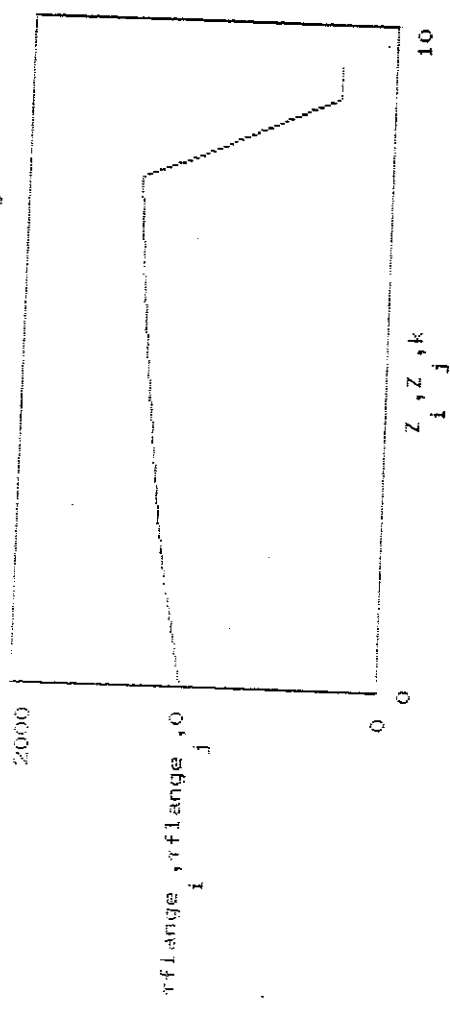


Torsional Shear Stresses Along Beam Length

psi



psi



$f_a = 3.561 \cdot 10^3$

psi (axial stress)

CASE 9

psi
Warping Normal
Stress
σ_{ws}

Position	z	σ _{ws}
0	0	5.836 · 10 ⁻³
1.272	1	4.993 · 10 ⁻³
2.543	1	4.153 · 10 ⁻³
3.815	2	3.317 · 10 ⁻³
5.087	2	1.653 · 10 ⁻²
6.358	2	8.228 · 10 ⁻²
7.63	2	-6.258
8.902	2	-5.475
9.96	2	-4.693
9.077	2	-3.911
9.136	2	-3.129
9.194	2	-2.347
9.253	2	-1.564
9.311	2	-0.782
9.37	2	-2.122 · 10 ⁻⁶

psi
Web Shear
Stress
τ_{tw}

Position	z	τ _{tw}
0	0	6.028 · 10 ⁻¹²
1.272	1	5.34 · 10 ⁻¹¹
2.543	1	9.849 · 10 ⁻¹¹
3.815	2	1.353 · 10 ⁻¹⁰
5.087	2	1.843 · 10 ⁻¹⁰
6.358	2	1.965 · 10 ⁻¹⁰
7.63	2	2.006 · 10 ⁻¹⁰
8.902	2	2.006 · 10 ⁻¹⁰
9.96	2	2.006 · 10 ⁻¹⁰
9.077	2	2.006 · 10 ⁻¹⁰
9.136	2	2.006 · 10 ⁻¹⁰
9.194	2	2.006 · 10 ⁻¹⁰
9.253	2	2.006 · 10 ⁻¹⁰
9.311	2	2.006 · 10 ⁻¹⁰
9.37	2	2.005 · 10 ⁻¹⁰
	2	2.005 · 10 ⁻¹⁰
	2	2.005 · 10 ⁻¹⁰

psi
Flange Shear
Stress
τ_{flange}

Position	z	τ _{flange}
0	0	1.082 · 10 ⁻³
1.272	3	1.164 · 10 ⁻³
2.543	3	1.233 · 10 ⁻³
3.815	3	1.289 · 10 ⁻³
5.087	3	1.363 · 10 ⁻³
6.358	3	1.382 · 10 ⁻³
7.63	2	3.057 · 10 ⁻²
8.902	2	3.057 · 10 ⁻²
9.96	2	3.057 · 10 ⁻²
9.077	2	3.057 · 10 ⁻²
9.136	2	3.057 · 10 ⁻²
9.194	2	3.057 · 10 ⁻²
9.253	2	3.057 · 10 ⁻²
9.311	2	3.057 · 10 ⁻²
9.37	2	3.057 · 10 ⁻²

Radians
Twist Angle
F

Position	z	F
0	0	1.269 · 10 ⁻⁵
1.272	1	4.821 · 10 ⁻⁵
2.543	1	1.027 · 10 ⁻⁴
3.815	1	1.724 · 10 ⁻⁴
5.087	1	2.534 · 10 ⁻⁴
6.358	1	3.419 · 10 ⁻⁴
7.63	1	4.343 · 10 ⁻⁴
8.902	1	4.386 · 10 ⁻⁴
9.96	1	4.428 · 10 ⁻⁴
9.077	1	4.471 · 10 ⁻⁴
9.136	1	4.514 · 10 ⁻⁴
9.194	1	4.557 · 10 ⁻⁴
9.253	1	4.6 · 10 ⁻⁴
9.311	1	4.642 · 10 ⁻⁴
9.37	1	4.685 · 10 ⁻⁴

Position

z

Position	z	τ _{flange}
0	0	1.082 · 10 ⁻³
1.272	1	1.164 · 10 ⁻³
2.543	1	1.233 · 10 ⁻³
3.815	1	1.289 · 10 ⁻³
5.087	1	1.363 · 10 ⁻³
6.358	1	1.382 · 10 ⁻³
7.63	1	3.057 · 10 ⁻²
8.902	1	3.057 · 10 ⁻²
9.96	1	3.057 · 10 ⁻²
9.077	1	3.057 · 10 ⁻²
9.136	1	3.057 · 10 ⁻²
9.194	1	3.057 · 10 ⁻²
9.253	1	3.057 · 10 ⁻²
9.311	1	3.057 · 10 ⁻²
9.37	1	3.057 · 10 ⁻²

$$E = 2.9 \cdot 10^7$$

Modulus of Elasticity

$$G = 1.12 \cdot 10^7$$

Modulus of Rigidity

$$P = 2.521 \cdot 10^4$$

Axial Load (lb)

$$M = 1.427 \cdot 10^4$$

Torsional Moment (in-lb)

Wide-Flange Constants (from AISC handbook)

$$J = 0.35$$

Torsional Constant

$$a = 4.4 \cdot 10^1$$

(ECw/GJ)

$$W_{no} = 1.22 \cdot 10^1$$

Normalized Warping Constant

$$S_w = 7.94$$

Warping Static Moment

$$A = 7.08$$

Area of Cross-Section

Wide-Flange Dimensions (inches)

$$L = 9.37$$

Unsupported Length of Beam

$$t_w = 0.245$$

Web Thickness

$$t_f = 0.4$$

Flange Thickness

$$h = 7.93$$

Overall Height of Wide-Flange

$$w = 6.495$$

Overall Width of Wide-Flange

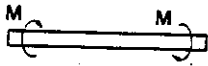
ERRATA

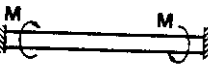
Torsional Analysis of Steel Members

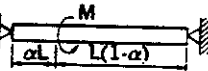
THE LAST TWO PAGES OF APPENDIX C WERE INADVERTENTLY OMITTED

SOLUTIONS TO DIFFERENTIAL EQUATIONS FOR VARIOUS LOADINGS AND BOUNDARY CONDITIONS

The following are the solutions of the differential equations using the proper boundary conditions. Take derivatives of ϕ equation to find ϕ' , ϕ'' , and ϕ''' .

CASE 1  $\phi = \frac{MZ}{GJ}$

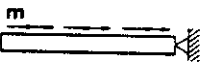
CASE 2  $\phi = \frac{Ma}{GJ} \left(\tanh \frac{L}{2a} \cdot \cosh \frac{Z}{a} - \tanh \frac{L}{2a} + \frac{Z}{a} - \sinh \frac{Z}{a} \right)$

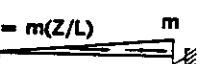
CASE 3  $0 \leq Z \leq \alpha L$

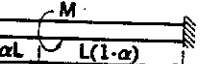
$$\phi = \frac{ML}{GJ} \left[(1.0 - \alpha) \frac{Z}{L} + \left(\frac{\sinh \frac{\alpha L}{a}}{\tanh \frac{L}{a}} - \cosh \frac{\alpha L}{a} \right) \cdot \frac{a}{L} \cdot \sinh \frac{Z}{a} \right]$$

$$\alpha L \leq Z \leq L$$

$$\phi = \frac{ML}{GJ} \left[(L - Z) \frac{\alpha}{L} + \frac{a}{L} \left(\frac{\sinh \frac{\alpha L}{a}}{\tanh \frac{L}{a}} \cdot \sinh \frac{Z}{a} - \sinh \frac{\alpha L}{a} \cdot \cosh \frac{Z}{a} \right) \right]$$

CASE 4  $\phi = \frac{ma^2}{GJ} \left[\frac{L^2}{2a^2} \left(\frac{Z}{L} - \frac{Z^2}{L^2} \right) + \cosh \frac{Z}{a} - \tanh \frac{L}{2a} \cdot \sinh \frac{Z}{a} - 1.0 \right]$

CASE 5  $M = m(Z/L)$ $\phi = \frac{mL^2}{GJ} \left[\frac{Z}{6L} - \frac{Z}{L} \cdot \frac{a^2}{L^2} + \frac{a^2}{L^2} \cdot \frac{\sinh \frac{Z}{a}}{\sinh \frac{L}{a}} - \frac{Z^3}{6L^3} \right]$

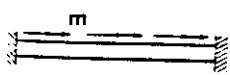
CASE 6  $0 \leq Z \leq \alpha L$

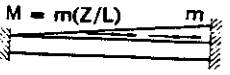
$$\phi = \frac{Ma}{(H+1)GJ} \left\{ \left[H \cdot \left(\frac{1}{\sinh \frac{L}{a}} + \sinh \frac{\alpha L}{a} - \frac{\cosh \frac{\alpha L}{a}}{\tanh \frac{L}{a}} \right) + \left(\sinh \frac{\alpha L}{a} - \frac{\cosh \frac{\alpha L}{a}}{\tanh \frac{L}{a}} + \frac{1}{\tanh \frac{L}{a}} \right) \right] \cdot \left[\cosh \frac{Z}{a} - 1.0 \right] - \sinh \frac{Z}{a} + \frac{Z}{a} \right\}$$

$$\alpha L \leq Z \leq L$$

$$\phi = \frac{Ma}{\left(1 + \frac{1}{H}\right)GJ} \left\{ \left[\frac{1}{H} \cdot \frac{1}{\sinh \frac{L}{a}} \left(\cosh \frac{\alpha L}{a} - 1.0 \right) + \frac{\left(\cosh \frac{\alpha L}{a} - \cosh \frac{L}{a} + \frac{L}{a} \cdot \sinh \frac{L}{a} \right)}{\sinh \frac{L}{a}} \right] + \cosh \frac{Z}{a} \left[\frac{1}{H} \cdot \frac{1}{\tanh \frac{L}{a}} \left(1.0 - \cosh \frac{\alpha L}{a} \right) + \frac{\left(1.0 - \cosh \frac{\alpha L}{a} \cdot \cosh \frac{L}{a} \right)}{\sinh \frac{L}{a}} \right] + \sinh \frac{Z}{a} \left[\frac{1}{H} \left(\cosh \frac{\alpha L}{a} - 1.0 \right) + \cosh \frac{\alpha L}{a} \right] - \frac{Z}{a} \right\}$$

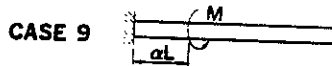
$$\text{Where: } H = \frac{\left[\frac{1}{\tanh \frac{L}{a}} \left(1.0 - \cosh \frac{\alpha L}{a} \right) + \frac{1}{\sinh \frac{L}{a}} \left(\cosh \frac{\alpha L}{a} - 1.0 \right) + \sinh \frac{\alpha L}{a} - \frac{\alpha L}{a} \right]}{\left[\frac{1}{\sinh \frac{L}{a}} \left(\cosh \frac{L}{a} + \cosh \frac{\alpha L}{a} \cdot \cosh \frac{L}{a} - \cosh \frac{\alpha L}{a} - 1.0 \right) + \frac{L}{a} (\alpha - 1.0) - \sinh \frac{\alpha L}{a} \right]}$$

CASE 7 
$$\phi = \frac{mLa}{2GJ} \left[\frac{1 + \cosh \frac{L}{a}}{\sinh \frac{L}{a}} \left(\cosh \frac{Z}{a} - 1.0 \right) + \frac{Z}{a} \left(1 - \frac{Z}{L} \right) - \sinh \frac{Z}{a} \right]$$

CASE 8 
$$\phi = \frac{mL^2}{GJ} \left\{ \left[\frac{a}{2L \sinh \frac{L}{a}} - S \cdot \tanh \frac{L}{2a} \right] \cdot \left(\cosh \frac{Z}{a} - 1.0 \right) + S \cdot \left(\sinh \frac{Z}{a} - \frac{Z}{a} \right) - \frac{Z^3}{6L^3} \right\}$$

Where:

$$S = \frac{\left(\cosh \frac{L}{a} - 1.0 \right) \cdot \frac{a}{2L} - \frac{\sinh \frac{L}{a}}{6.0}}{\left(\frac{L}{a} \sinh \frac{L}{a} + 2.0 - 2 \cosh \frac{L}{a} \right)}$$

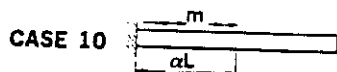


$0 \leq Z \leq \alpha L$

$$\phi = \frac{Ma}{GJ} \left[\left(\sinh \frac{\alpha L}{a} - \tanh \frac{L}{a} \cdot \cosh \frac{\alpha L}{a} + \tanh \frac{L}{a} \right) \left(\cosh \frac{Z}{a} - 1.0 \right) - \sinh \frac{Z}{a} + \frac{Z}{a} \right]$$

$\alpha L \leq Z \leq L$

$$\phi = \frac{Ma}{GJ} \left[\left(\tanh \frac{L}{a} \cdot \cosh \frac{\alpha L}{a} - \tanh \frac{L}{a} - \sinh \frac{\alpha L}{a} \right) - \left(\cosh \frac{\alpha L}{a} - 1.0 \right) \left(\tanh \frac{L}{a} \cdot \cosh \frac{Z}{a} \right) + \left(\cosh \frac{\alpha L}{a} - 1.0 \right) \cdot \sinh \frac{Z}{a} + \frac{\alpha L}{a} \right]$$

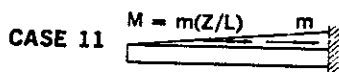


$0 \leq Z \leq \alpha L$

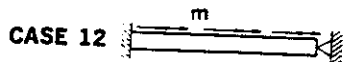
$$\phi = \frac{ma^2}{GJ} \left\{ \left[\tanh \frac{L}{a} \left(\frac{\alpha L}{a} - \sinh \frac{\alpha L}{a} \right) + \cosh \frac{\alpha L}{a} \right] \left[\cosh \frac{Z}{a} - 1.0 \right] - \frac{\alpha L}{a} \cdot \sinh \frac{Z}{a} + \frac{Z}{a} \left(\frac{\alpha L}{a} - \frac{Z}{2a} \right) \right\}$$

$\alpha L \leq Z \leq L$

$$\phi = \frac{ma^2}{GJ} \left\{ \tanh \left[\frac{L}{a} \right] \cdot \sinh \left[\frac{\alpha L}{a} \right] - \cosh \left[\frac{\alpha L}{a} \right] - \frac{\alpha L}{a} \cdot \tanh \left[\frac{L}{a} \right] + 1.0 + \frac{(\alpha^2 L^2)}{(2a^2)} - \left(\sinh \left[\frac{\alpha L}{a} \right] - \frac{\alpha L}{a} \right) \cdot \tanh \left[\frac{L}{a} \right] \cdot \cosh \left[\frac{Z}{a} \right] + \left(\sinh \left[\frac{\alpha L}{a} \right] - \frac{\alpha L}{a} \right) \cdot \sinh \left[\frac{Z}{a} \right] \right\}$$



$$\phi = \frac{ma^2}{GJ} \left\{ \left[-\frac{5}{6} \frac{L^2}{a^2} - \left(\frac{a}{L} - \frac{L}{2a} \right) \cdot \tanh \frac{L}{a} + 1.0 \right] + \left[-\frac{Z}{L} + \frac{ZL}{a^2} \right] + \left[\frac{a}{L} - \frac{L}{2a} \right] \left(\frac{\sinh \frac{Z}{a}}{\cosh \frac{L}{a}} \right) - \frac{Z^2}{6a^2} \cdot \frac{Z}{L} \right\}$$



$$\phi = \frac{ma^2}{GJ} \left[H \cdot \left(\tanh \frac{L}{a} - \frac{Z}{a} - \tanh \frac{L}{a} \cdot \cosh \frac{Z}{a} + \sinh \frac{Z}{a} \right) + \frac{\cosh \frac{Z}{a}}{\cosh \frac{L}{a}} - \frac{1}{\cosh \frac{L}{a}} - \frac{Z^2}{2a^2} \right]$$

Where:

$$H = \left[\frac{L^2}{2a^2} - 1.0 + \frac{1}{\cosh \frac{L}{a}} \right] \cdot \frac{1}{\left(\tanh \frac{L}{a} - \frac{L}{a} \right)}$$

Figure 17, pg. 17. Span should be 25 ft; Left reaction should be 231 kips.

